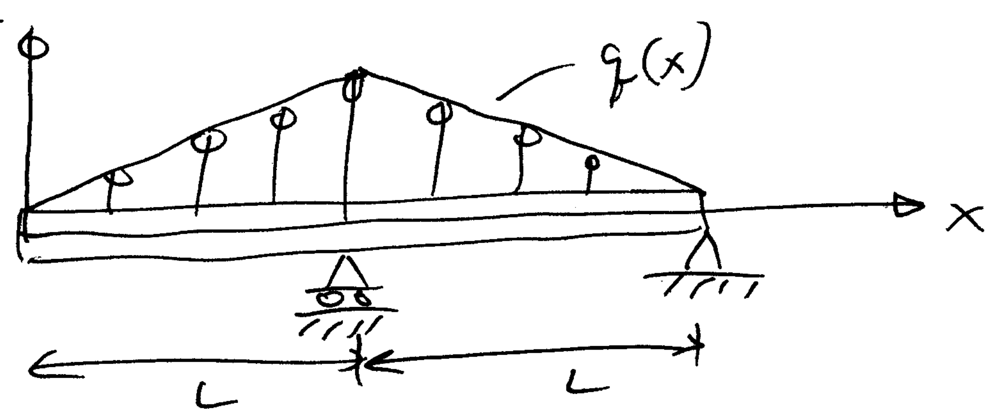
PAL
2/14/08

UNIFIED ENGINEERING
Problem Set - Week 3 Spring 2008

SOLUTIONS

M3.1



First determine the relative magnitude of q(x) in terms of P.

We are given:

$$\int_0^{2L} q(x)\,dx = P$$

We need a functional expression for q(x). Assign it a value of q_0 at the midpoint ($x = L$) and work to determine that value. Note two "parts" to q(x): $0<x<L$; $L<x<2L$. Consider each of these separately.

For $0 < x < L$, $q(x)$ increases linearly from a value of 0 at $x = 0$ to q_0 at $x = L$. So:

$$q(x) = \frac{q_0}{L} x \qquad 0 < x < L$$

Check:

$$q(0) = 0 \checkmark$$

$$q(L) = q_0 \checkmark$$

$$\frac{dq(x)}{dx} = \frac{q_0}{L} \quad \text{increases linearly} \checkmark$$

Now for $L < x < 2L$, $q(x)$ decreases linearly from a value of q_0 at $x = L$ to a value of 0 at $x = 2L$. So:

$$q(x) = q_0 \left(\frac{2L - x}{L} \right)$$

(NOTE: can determine by beginning with $q(x) = ax + b$ and using the two conditions to get two equations in a and b and solve for a and b)

Check: (good engineers always do when they can)

$$q(L) = q_0 \checkmark$$

$$q(2L) = 0 \checkmark$$

$$\frac{dq(x)}{dx} = -\frac{q_0}{L} \quad \text{decreases linearly} \checkmark$$

Now determine q_0. Remember:

$$\int_0^{2L} q(x)\,dx = P$$

$$\Rightarrow \int_0^L q(x)\,dx + \int_L^{2L} q(x)\,dx = P$$

$$\int_0^L \frac{q_0}{L} x\,dx + \int_L^{2L} \frac{q_0}{L}(2L-x)\,dx = P$$

$$\left.\frac{q_0 x^2}{2L}\right]_0^L + \left.\left(2q_0 x - \frac{q_0 x^2}{2L}\right)\right]_L^{2L} = P$$

$$\frac{q_0 L}{2} + q_0\left[4L - 2L - \left(2L - \frac{L}{2}\right)\right] = P$$

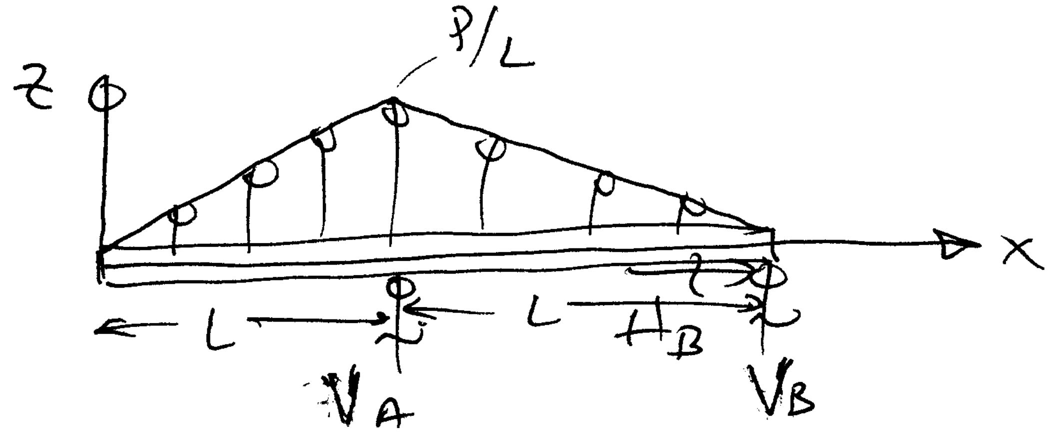
$$\Rightarrow \quad q_0 L = P \quad \text{giving} \quad \boxed{q_0 = \frac{P}{L}}$$

Finally:

$$q(x) = \begin{cases} \dfrac{Px}{L^2} & 0 < x < L \\ \dfrac{P}{L^2}(2L - x) & L < x < 2L \end{cases}$$

Proceed to:

(a) First step is to draw the Free Body Diagram.



Use equilibrium:

$$\sum F_x = 0 \quad \xrightarrow{+} \quad \Rightarrow H_B = 0$$

$$\sum F_z = 0 \quad \uparrow + \quad \Rightarrow V_A + V_B + \underbrace{\int_0^{2L} q_0(x) = 0}_{\text{Recall this integrates to a magnitude of } P}$$

$$\Rightarrow V_A + V_B = -P \qquad (1)$$

$$\sum M_A = 0 \quad \circlearrowright + \quad \Rightarrow V_B(L) - \int_0^L q_0(x)(L-x)\,dx + \int_L^{2L} q_0(x)(x-L)\,dx = 0$$

$$\Rightarrow V_B L - \int_0^L \frac{Px}{L^2}(L-x)\,dx + \int_L^{2L} \frac{P}{L^2}(2L-x)(x-L)\,dx = 0$$

$$V_B L - \left(\frac{Px^2}{2L} - \frac{Px^3}{3L^2}\right)\Bigg]_0^L + \int_L^{2L} \frac{P}{L^2}\left(-2L^2 + 3Lx - x^2\right)dx = 0$$

$$V_B L - \frac{PL}{2} + \frac{PL}{3} + \frac{P}{L^2}\left[-2L^2 x + \frac{3}{2}Lx^2 - \frac{x^3}{3}\right]_L^{2L} = 0$$

Continuing:

$$V_B L - \frac{PL}{6} + \frac{P}{L^2}\left[-4L^3 + 6L^3 - \frac{8}{3}L^3 - \left(-2L^3 + \frac{3}{2}L^3 - \frac{L^3}{3}\right)\right] = 0$$

$$\Rightarrow V_B L - PL\left(\frac{1}{6} - 4 + 6 - \frac{8}{3} + 2 - \frac{3}{2} + \frac{1}{3}\right) = 0$$

$$V_B = P\left(4 - \frac{1}{6} - \frac{14}{6} - \frac{9}{6}\right)$$

$$\Rightarrow \underline{\underline{V_B = 0}}$$

Using (1): $V_A = -P$

Summarizing, the reactions are:

$$\boxed{\begin{aligned} H_B &= 0 \\ V_A &= -P \\ V_B &= 0 \end{aligned}}$$

<u>Note 1</u>: The integral in the moment for $q(x)$ could have been found to be zero about A by inspection due to the symmetry of $q(x)$ about $x = L$. The $q(x)$ to the left counters that to the right.

Note 2: It appears that the beam "teeters" on the roller support. However the pin provides a reaction if there is any shift in $q(x)$.

(b) This needs to be done in parts since there is a point load (reaction) along the beam at $x = L$ and there is also a change in $q(x)$ also at $x = L$.

→ So, for $0 < x < L$:

There is no loading in x ⇒ $\boxed{F(x) = 0}$

Found:

$$q(x) = \frac{Px}{L^2}$$

Use $\frac{dS}{dx} = q(x)$

$$\Rightarrow S(x) = \int q(x)\,dx = \int \frac{Px}{L^2}\,dx$$

$$\Rightarrow S(x) = \frac{Px^2}{2L^2} + C_1$$

Use a boundary condition to get the constant of integration.

Look at $x = 0^+$:

$\downarrow S(0)^+$ so: $\sum F_z = 0 \uparrow + \Rightarrow 0 = S(0^+)$

no other loads

giving: $S(0^+) = C_1 \Rightarrow C_1 = 0$

finally yielding: $S(x) = \dfrac{Px^2}{2L^2}$

Proceeding to the moment...

$$\frac{dM}{dx} = S$$

$$\Rightarrow M(x) = \int S(x)\,dx = \int \frac{Px^2}{2L^2}\,dx$$

$$= \frac{Px^3}{6L^2} + C_2$$

Again, use a boundary condition. At $x=0$ there is no applied moment so $M(0^+) = 0$

giving: $C_2 = 0$

$$\Rightarrow M(x) = \frac{Px^3}{6L^2}$$

Summarizing: for $0<x<L$

$$\boxed{\begin{aligned} q(x) &= \frac{Px}{L^2} \\ P(x) &= 0 \\ S(x) &= \frac{Px^2}{2L^2} \\ M(x) &= \frac{Px^3}{6L^2} \end{aligned}}$$

→ Move on to $\underline{L < x < 2L}$:

There is still no loading in x, so: $\boxed{F(x) = 0}$

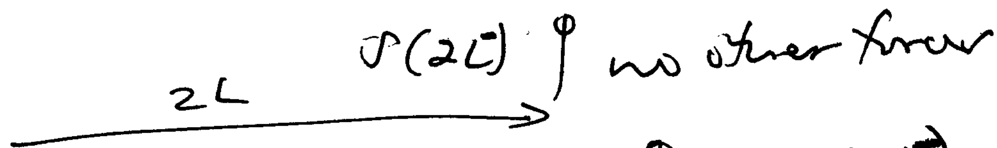
using $\frac{dS(x)}{dx} = q(x)$ with $q(x) = \frac{P}{L^2}(2L - x)$

we again get:

$$S(x) = \int q(x)\,dx = \int \frac{P}{L^2}(2L - x)\,dx$$

$$\Rightarrow S(x) = \frac{2Px}{L} - \frac{Px^2}{2L^2} + C_3$$

There are differences in the boundary conditions in this sector, but one can go to the tip ($x = 2L$) and take a cut giving a "negative" force. There is no reactions so:



$$\sum F(z) = 0 \quad \uparrow+ \Rightarrow S(2L^-) = 0$$

giving $S(2L) = 0 = 4P - 2P + C_3$

$$\Rightarrow C_3 = -2P$$

$$\Rightarrow S(x) = \frac{P}{L}\left(2x - \frac{x^2}{2L} - 2L\right)$$

Proceed again to $\frac{dM}{dx} = S(x)$

$$\Rightarrow M(x) = \int S(x)\,dx = \int \frac{P}{L}\left(2x - \frac{x^2}{2L} - 2L\right)dx$$

$$\Rightarrow M(x) = \frac{Px^2}{L} - \frac{Px^3}{6L^2} - 2Px + C_4$$

Again going to the tip ($x = 2L$), there are no applied or reactionary moments, so

$$M(2L^-) = 0$$

$$\Rightarrow 0 = 4PL - \frac{4}{3}PL - 4PL + C_4$$

$$\Rightarrow C_4 = \frac{4}{3}PL$$

Finally: $M(x) = P\left(\frac{x^2}{L} - \frac{x^3}{6L^2} - 2x + \frac{4}{3}L\right)$

Summarizing: for $L < x < 2L$

$$\boxed{\begin{aligned} q(x) &= \frac{P}{L^2}(2L - x) \\ F(x) &= 0 \\ S(x) &= \frac{P}{L}\left(2x - \frac{x^2}{2L} - 2L\right) \\ M(x) &= P\left(\frac{x^2}{L} - \frac{x^3}{6L^2} - 2x + \frac{4}{3}L\right) \end{aligned}}$$

→ A way to check -- there are no point moments applied, so the solutions for $M(x)$ for the two segments must be equal at $x = L$:

$$\frac{PL}{6} \stackrel{?}{=} P\left(L - \frac{L}{6} - 2L + \frac{4}{3}L\right)$$

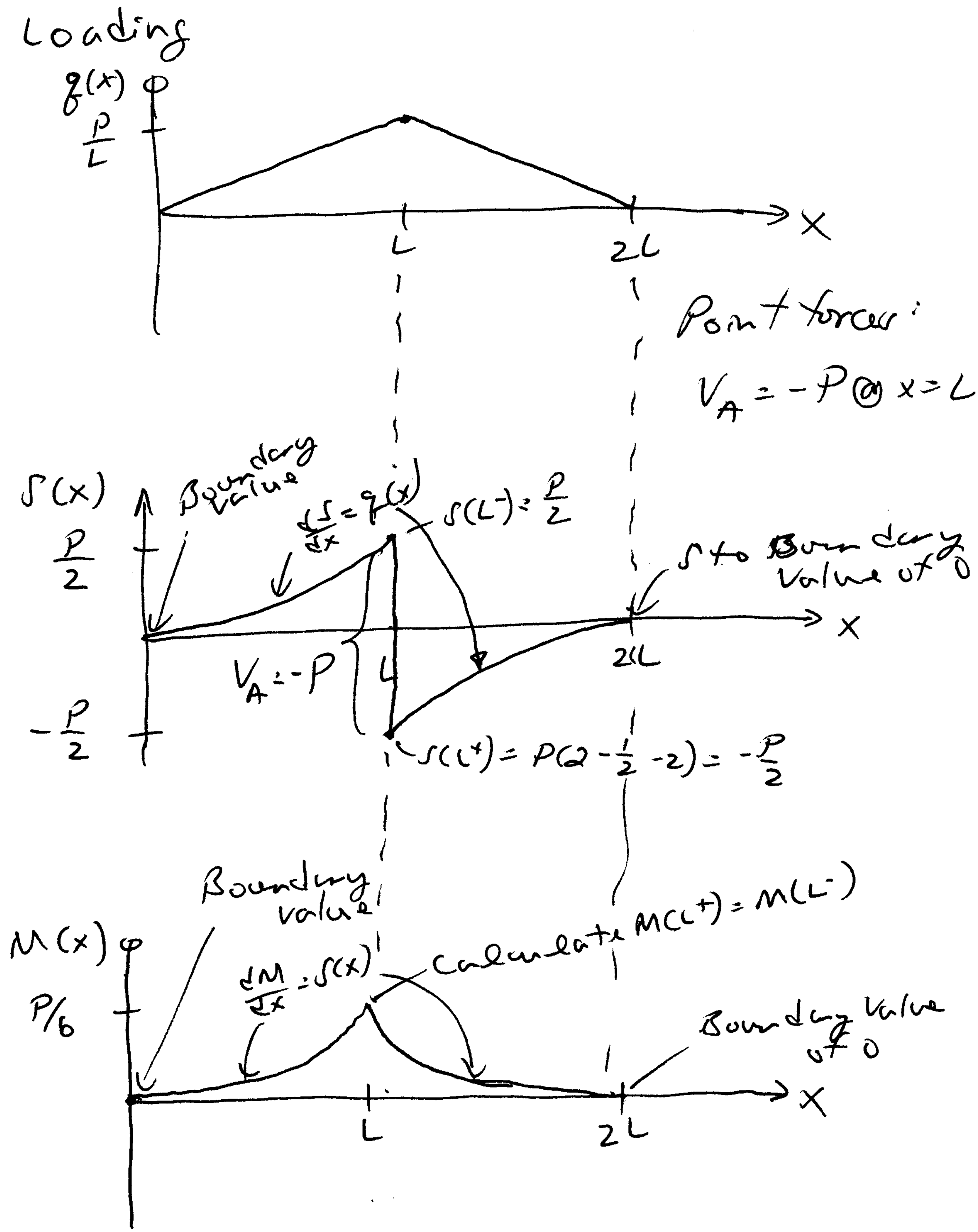
$$\frac{PL}{6} \stackrel{?}{=} P\left(-\frac{L}{6} - L + \frac{8}{6}L\right)$$

$$\frac{PL}{6} = \frac{PL}{6} \quad \checkmark \qquad \underline{\underline{YES}}$$

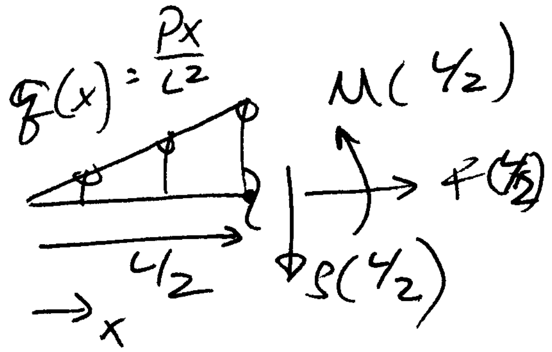
→ Now draw these. In sketching, use the relations of the derivatives to get a shape. Calculate end point values to begin. And recall that point loads cause equal jumps in shear (account for proper direction and sign).

$F(x) = 0$ everywhere.... no need to plot

to next page...


Loading
$q(x)$
$\frac{P}{L}$
L
$2L$
x
Point forces:
$V_A = -P$ @ $x = L$
$S(x)$
Boundary Value
$\frac{dS}{dx} = q(x)$
$S(L^-) = \frac{P}{2}$
$\frac{P}{2}$
S to Boundary Value of 0
$V_A = -P$
L
$2L$
x
$-\frac{P}{2}$
$S(L^+) = P(2 - \frac{1}{2} - 2) = -\frac{P}{2}$
Boundary value
$M(x)$
$P/6$
$\frac{dM}{dx} = S(x)$
Calculate $M(L^+) = M(L^-)$
Boundary Value of 0
L
$2L$
x

(c) Cut the beam, first at $x = L/2$



Use equilibrium:

$\sum F_x = 0 \quad \xrightarrow{+} \quad \Rightarrow \quad F(L/2) = 0$ ✓ checks

$\sum F_z = 0 \quad \uparrow + \quad \Rightarrow \quad \int_0^{L/2} \frac{Px}{L^2}\,dx - S(L/2) = 0$

$$\Rightarrow S(L/2) = \left.\frac{Px^2}{2L^2}\right]_0^{L/2}$$

$$S(L/2) = \frac{P}{8}$$

Check: $S(x) = \frac{Px^2}{2L^2} \Rightarrow S(L/2) = \frac{P}{8}$ ✓

Finally:

$\sum M_{L/2} = 0 \quad \circlearrowleft + \Rightarrow M(L/2) - \int_0^{L/2} \frac{Px}{L^2}\left(L/2 - x\right)dx = 0$

$$\Rightarrow M(L/2) = \left.\frac{Px^2}{4L} - \frac{Px^3}{3L^2}\right]_0^{L/2}$$

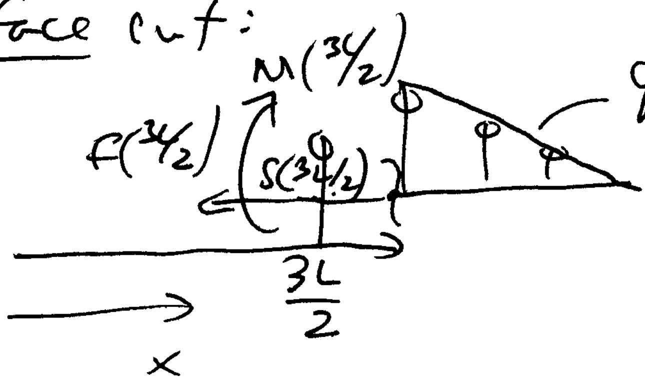
$$= \frac{PL}{16} - \frac{PL}{24} = PL\left[\frac{3}{48} - \frac{2}{48}\right] = \frac{PL}{48}$$

Check: $M(L/2) = \dfrac{PL}{48}$ ✓

→ Now cut the beam at $x = \dfrac{3L}{2}$

Make it simpler by taking a <u>negative face</u> cut:

$q(x) = \dfrac{P}{L^2}(2L - x)$



Again use equilibrium:

$\sum F_x = 0 \;\xrightarrow{+}\; \Rightarrow F\left(\dfrac{3L}{2}\right) = 0$ ✓

$\sum F_z = 0 \;\uparrow + \;\Rightarrow S\left(\dfrac{3L}{2}\right) + \displaystyle\int_{3L/2}^{2L} \dfrac{P}{L^2}(2L - x)\,dx = 0$

$$\Rightarrow S\left(\frac{3L}{2}\right) = -\frac{2Px}{L} + \frac{Px^2}{2L^2}\Bigg]_{3L/2}^{2L}$$

$$S\left(\frac{3L}{2}\right) = -4P + 2P + 3P - \frac{9P}{8}$$

giving $S\left(\dfrac{3L}{2}\right) = -\dfrac{P}{8}$

Check: $S(x) = \dfrac{P}{L}\left(2x - \dfrac{x^2}{2L} - 2L\right)$

$\Rightarrow S\left(\dfrac{3L}{2}\right) = P\left(3 - \dfrac{9}{8} - 2\right) = -\dfrac{P}{8}$ ✓

$$\sum M_{3L/2} = 0 \quad \circlearrowleft + \Rightarrow -M\left(\frac{3L}{2}\right) + \int_{3L/2}^{2L} \frac{P}{L^2}(2L-x)\left(x-\frac{3L}{2}\right)dx = 0$$

$$\Rightarrow M\left(\frac{3L}{2}\right) = \int_{3L/2}^{2L} \frac{P}{L^2}\left(-3L^2 + \frac{7}{2}Lx - x^2\right)dx$$

$$M\left(\frac{3L}{2}\right) = -3Px + \frac{7P}{4L}x^2 - \frac{P}{3L^2}x^3 \Big]_{3L/2}^{2L}$$

$$= PL\left(-6 + 7 - \frac{8}{3} + \frac{9}{2} - \frac{63}{16} + \frac{9}{4}\right)$$

$$= PL\left(\frac{48}{48} - \frac{128}{48} + \frac{216}{48} - \frac{189}{48} + \frac{108}{48}\right)$$

$$\Rightarrow M\left(\frac{3L}{2}\right) = \frac{PL}{48}$$

Check:

$$M(x) = P\left(\frac{x^2}{L} - \frac{x^3}{6L^2} - 2x + \frac{4}{3}L\right)$$

$$\Rightarrow M\left(\frac{3L}{2}\right) = PL\left(\frac{9}{4} - \frac{9}{16} - 3 + \frac{4}{3}\right)$$

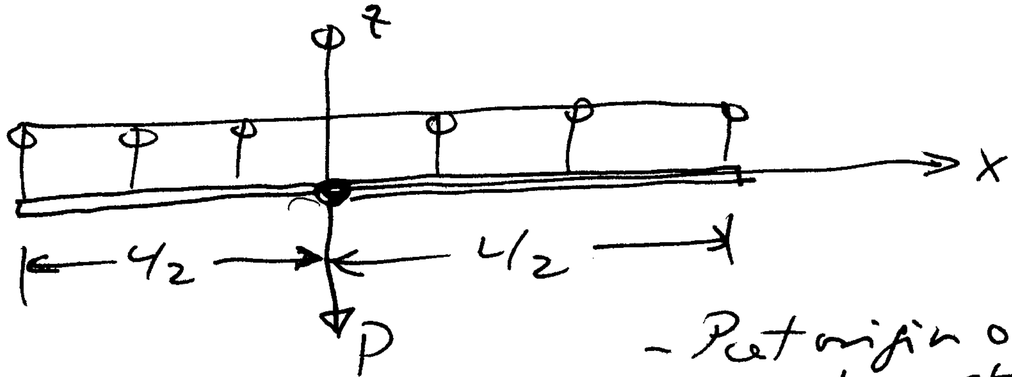
$$= PL\left(\frac{108}{48} - \frac{27}{48} - \frac{144}{48} + \frac{64}{48}\right)$$

finally: $M\left(\frac{3L}{2}\right) = \frac{PL}{48}$ ✓

ALL CHECK

M 3.2

Model for Case 1: Lift constant along wing span



– Put origin of x-z system at fuselage

(a) Note that this can be done irrespective of the model used. Consider the rear ones .....

The model is the free body diagram. There are no reaction forces since the wing has no internal supports that can carry the load.

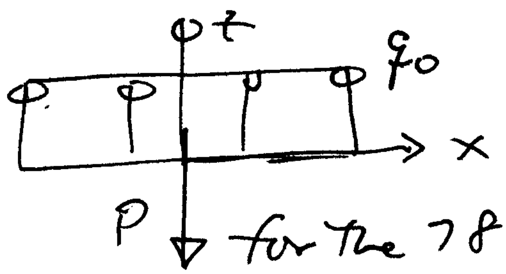
NOTE: It is not dynamic because of symmetry of the lift (for all models) resulting in a moment balance and because of the special condition that the integrated lift force equals total plane weight in stable flight (for all cases).

Further NOTE: If the lift is not symmetric, we get rotation about the fuselage -- a way to maneuver the aircraft.

(b) We now must consider this case by case.

→ Case 1 -- lift constant along span

From last term:



$q(x) = q_0$

for the 787: $q_0 = 5400$ lb/ft

There are no axial forces, so $F(x) = 0$

We have two sections of the wing:

$$0 < x < L/2$$
$$-L/2 < x < 0$$

There is symmetry, so our results should be the same, but let's be sure:

For: $0 < x < L/2$

$$q(x) = q_0$$

use: $\frac{dS}{dx} = q(x) \Rightarrow S(x) = \int q_0 \, dx$

$$= q_0 x + C_1$$

Go to the tip and see $S = 0$ at $x = L/2$

$$\Rightarrow S(L/2) = 0 = \frac{q_0 L}{2} + C_1 \Rightarrow C_1 = -\frac{q_0 L}{2}$$

$$\Rightarrow S(x) = q_0 \left(x - \frac{L}{2}\right)$$

Proprerto:

$$\frac{dM}{dx} = S$$

$$\Rightarrow M(x) = \int q_0 \left(x - \frac{L}{2}\right) dx$$

$$= q_0 \frac{x^2}{2} - q_0 \frac{L}{2} x + C_2$$

Again, at the tip: $M = 0$ at $x = L/2$

So: $M(L/2) = q_0 \frac{L^2}{8} - q_0 \frac{L^2}{4} + C_2$

$$\Rightarrow C_2 = \frac{q_0 L^2}{8}$$

Finally: $M(x) = \frac{q_0}{2} \left(x^2 - Lx + \frac{L^2}{4}\right)$

Now for: $\underline{-L/2 < x < 0}$

again: $q(x) = q_0$

using: $\frac{dS}{dx} = q(x) \Rightarrow S(x) = \int q_0\, dx$

$$= q_0 x + C_3$$

Go to that tip ($x = -L/2$) where $S = 0$ and:

$$S(-L/2) = -q_0 L/2 + C_3 \Rightarrow C_3 = \frac{q_0 L}{2}$$

$$\Rightarrow S(x) = q_0 \left(x + \frac{L}{2}\right)$$

Note that the value must change by the concentrated load of the weight P at the root ($\frac{dS}{dx}(x=0) = -P$) and it does as $P = q_0 L$!

and finally with: $\frac{dM}{dx} = S(x)$

$$\Rightarrow M(x) = \int q_0 \left(x + \frac{L}{2}\right) dx$$

$$= q_0 \frac{x^2}{2} + q_0 \frac{L}{2} x + C_4$$

Again, at this tip ($x = -L/2$), the moment is zero. So:

$$M(-L/2) = 0 = \frac{q_0 L^2}{8} - q_0 \frac{L^2}{4} + C_4$$

$$\Rightarrow C_4 = \frac{q_0 L^2}{8}$$

resulting in: $M(x) = \frac{q_0}{2}\left(x^2 + Lx + \frac{L^2}{4}\right)$

This is symmetric about the root as x switches from + to − in value.

Summarizing for Case 1

$$\boxed{\begin{aligned} F(x) &= 0 \quad \text{everywhere} \\ S(x) &= q_0\,(x - L/2) \quad 0 < x < L/2 \\ &= q_0\,(x + L/2) \quad -L/2 < x < 0 \\ M(x) &= \frac{q_0}{2}\left(x^2 - Lx + \frac{L^2}{4}\right) \quad 0 < x < L/2 \\ &= \frac{q_0}{2}\left(x^2 + Lx + \frac{L^2}{4}\right) \quad -L/2 < x < 0 \\ &\text{with: } q_0 = 2700\,\tfrac{\text{lbs}}{\text{ft}}, \quad L = 200\text{ ft} \end{aligned}}$$

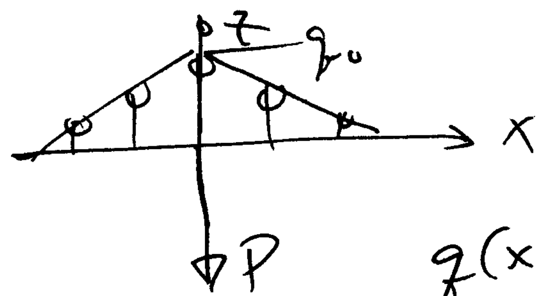
Note concerning symmetry: We showed the symmetry in this case. For subsequent cases, we use this symmetry and calculate for only one wing and determine the expression for the other wing by:

(1) changing $S(x)$ by P at the fuselage ($+P$ if proceeding from $+x$ to $-x$) and changing the sign of even powers of x

(2) changing odd powers of x in the moment expression in their sign

→ Case 2: lift linear variation along span from zero at tip to maximum value at root

From last term:



$$q(x) = q_0\left(1 - \frac{2x}{L}\right) \quad 0 < x < \frac{L}{2}$$

$$= q_0\left(1 + \frac{2x}{L}\right) \quad -L/2 < x < 0$$

For the 787: $q_0 = 5400$ lbs/ft, $L = 100$ ft

Again, there are no axial forces so $F(x) = 0$

Use $\frac{dS}{dx} = q(x) \Rightarrow S(x) = \int q(x)$ for $0 < x < L/2$:

$$S(x) = \int q_0\left(1 - \frac{2x}{L}\right) dx$$

$$= q_0 x - q_0 \frac{x^2}{L} + C_1$$

In all cases, we have at the tip $S(L/2) = 0$

So: $q_0 \frac{L}{2} - q_0 \frac{L}{4} + C_1 = 0$

$$\Rightarrow C_1 = -\frac{q_0 L}{4}$$

$$\Rightarrow S(x) = q_0\left(x - \frac{x^2}{L} - \frac{L}{4}\right)$$

Progressing to:

$$\frac{dM}{dx} = S$$

$$\Rightarrow M(x) = \int q_0 \left(x - \frac{x^2}{L} - \frac{L}{4}\right) dx$$

$$= q_0 \frac{x^2}{2} - q_0 \frac{x^3}{3L} - q_0 \frac{Lx}{4} + C_2$$

Again at the tip $(x = L/2)$ $M = 0$

$$\Rightarrow M\left(\frac{L}{2}\right) = 0 = q_0 L^2 \left(\frac{1}{8} - \frac{1}{24} - \frac{1}{8}\right) + C_2$$

$$\Rightarrow C_2 = -q_0 L^2 \left(\frac{6}{48} - \frac{1}{24} - \frac{6}{48}\right)$$

giving: $C_2 = \frac{q_0 L^2}{24}$

Finally:

$$M(x) = q_0\left(\frac{x^2}{2} - \frac{x^3}{3L} - \frac{Lx}{4} + \frac{L^2}{24}\right)$$

Summarizing for Case 2

$$F(x) = 0 \quad \text{everywhere}$$

$$S(x) = q_0\left(x - \frac{x^2}{L} - \frac{L}{4}\right) \quad 0 < x < L/2$$

$$q_0\left(x + \frac{x^2}{L} - \frac{L}{4}\right) + P \quad -L/2 < x < 0$$

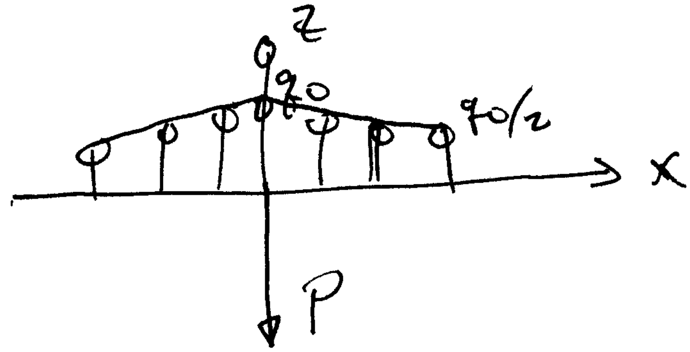
$$M(x) = q_0\left(\frac{x^2}{2} - \frac{x^3}{3L} - \frac{Lx}{4} + \frac{L^2}{24}\right) \quad 0 < x < L/2$$

$$= q_0\left(\frac{x^2}{2} + \frac{x^3}{3L} + \frac{Lx}{4} + \frac{L^2}{24}\right) \quad -L/2 < x < 0$$

with $q_0 = 5400$ lb/ft, $L = 200$ ft, $P = 540{,}000$ lbw

→ Case 3: lift linear variation along span with maximum value at root down to half this value at tip

From last term:



$$q(x) = q_0\left(1 - \frac{x}{L}\right) \qquad 0 < x < \frac{L}{2}$$
$$= q_0\left(1 + \frac{x}{L}\right) \qquad -L/2 < x < 0$$

for the 787 $q_0 = 8000$ lbs/ft, $L = 200$ ft

Again, there are no axial forces so $F(x) = 0$

Proceeding to $\frac{dS}{dx} = q(x) \Rightarrow S(x) = \int q(x)$

for $0 < x < L/2$

$$S(x) = \int q_0\left(1 - \frac{x}{L}\right)dx$$
$$= q_0 x - q_0 \frac{x^2}{2L} + C_1$$

Applying the condition of $S = 0$ at the tip $(x = L/2)$:

$$q_0 \frac{L}{2} - q_0 \frac{L}{8} + C_1 = 0$$
$$\Rightarrow C_1 = -\frac{3q_0 L}{8}$$
$$\Rightarrow S(x) = q_0\left(x - \frac{x^2}{2L} - \frac{3L}{8}\right)$$

Progressing to: $\frac{dM}{dx} = S$

$$\Rightarrow M(x) = \int q_0\left(x - \frac{x^2}{2L} - \frac{3L}{8}\right)dx$$

giving: $M(x) = \int q_0 \left(x - \frac{x^2}{2L} - \frac{3L}{8}\right) dx$

$$= q_0 \frac{x^2}{2} - q_0 \frac{x^3}{6L} - q_0 \frac{3Lx}{8} + C_2$$

Applying the condition of $M = 0$ at the top ($x = L/2$):

$$M\left(\tfrac{L}{2}\right) = 0 = q_0 L^2 \left(\frac{1}{8} - \frac{1}{48} - \frac{3}{16}\right) + C_2$$

$$\Rightarrow C_2 = -q_0 L^2 \left(\frac{6}{48} - \frac{1}{48} - \frac{9}{48}\right)$$

giving: $C_2 = \frac{q_0 L^2}{12}$

Finally: $M(x) = q_0 \left(\frac{x^2}{2} - \frac{x^3}{6L} - \frac{3Lx}{8} + \frac{L^2}{12}\right)$

Summarizing for <u>Case 3</u>

$F(x) = 0$ everywhere

$S(x) = q_0 \left(x - \frac{x^2}{2L} - \frac{3L}{8}\right) \qquad 0 < x < \frac{L}{2}$

$\quad = q_0 \left(x + \frac{x^2}{2L} - \frac{3L}{8}\right) + P \qquad -\frac{L}{2} < x < 0$

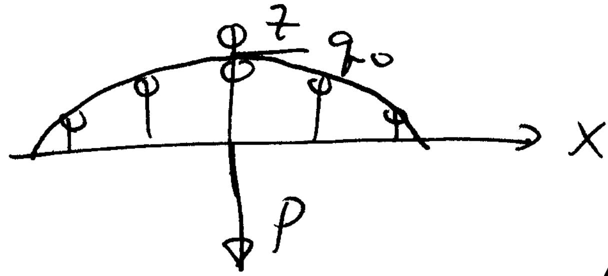
$M(x) = q_0 \left(\frac{x^2}{2} - \frac{x^3}{6L} - \frac{3Lx}{8} + \frac{L^2}{12}\right) \qquad 0 < x < L/2$

$\quad = q_0 \left(\frac{x^2}{2} + \frac{x^3}{6L} + \frac{3Lx}{8} + \frac{L^2}{12}\right) \qquad -L/2 < x < 0$

with $q_0 = 3600$ lbs/ft, $L = 200$ ft,
$P = 540\,000$ lbs.

→ Case 4: lift quadratic along span with maximum at root to zero at tip

From last term:



$$q(x) = q_0\left(1 - \frac{4x^2}{L^2}\right)$$

valid throughout

for the 787 $q_0 = 4048$ lbs/ft, $L = 200$ ft

There being no axial forces, $f(x) = 0$

Then with $\frac{dS}{dx} = q(x)$

$$\Rightarrow S(x) = \int q_0\left(1 - \frac{4x^2}{L^2}\right)dx$$

$$= q_0\left(x - \frac{4x^3}{3L^2}\right) + C_1$$

Using the condition of no shear at the tip:

$$S(L/2) = 0 = q_0\left(\frac{L}{2} - \frac{L}{6}\right) + C_1$$

$$\Rightarrow C_1 = -\frac{q_0 L}{3}$$

$$\Rightarrow S(x) = q_0\left(x - \frac{4x^3}{3L^2} - \frac{L}{3}\right)$$

And then to: $\dfrac{dM}{dx} = S$

$$\Rightarrow M(x) = \int q_0 \left(x - \frac{4x^3}{3L^2} - \frac{L}{3}\right) dx$$

$$= q_0 \left(\frac{x^2}{2} - \frac{x^4}{3L^2} - \frac{Lx}{3}\right) + C_2$$

And again using the condition of no moment at the tip:

$$M(L/2) = 0 = q_0 \left(\frac{L^2}{8} - \frac{L^2}{48} - \frac{L^2}{6}\right) + C_2$$

$$\Rightarrow C_2 = -q_0 L^2 \left(\frac{6}{48} - \frac{1}{48} - \frac{8}{48}\right)$$

$$\text{So:} \quad C_2 = \frac{q_0 L^2}{16}$$

Finally: $$M(x) = q_0 \left(\frac{x^2}{2} - \frac{x^4}{3L^2} - \frac{Lx}{3} + \frac{L^2}{16}\right)$$

Summarizing for <u>Case 4</u>

$F(x) = 0$ everywhere

$$S(x) = q_0 \left(x - \frac{4x^3}{3L^2} - \frac{L}{3}\right) \qquad 0 < x < L/2$$

$$= q_0 \left(x - \frac{4x^3}{3L^2} - \frac{L}{3}\right) + P \qquad -L/2 < x < 0$$

$$M(x) = q_0 \left(\frac{x^2}{2} - \frac{x^4}{3L^2} - \frac{Lx}{3} + \frac{L^2}{16}\right) \qquad 0 < x < L/2$$

$$= q_0 \left(\frac{x^2}{2} - \frac{x^4}{3L^2} + \frac{Lx}{3} + \frac{L^2}{16}\right) \qquad -L/2 < x < 0$$

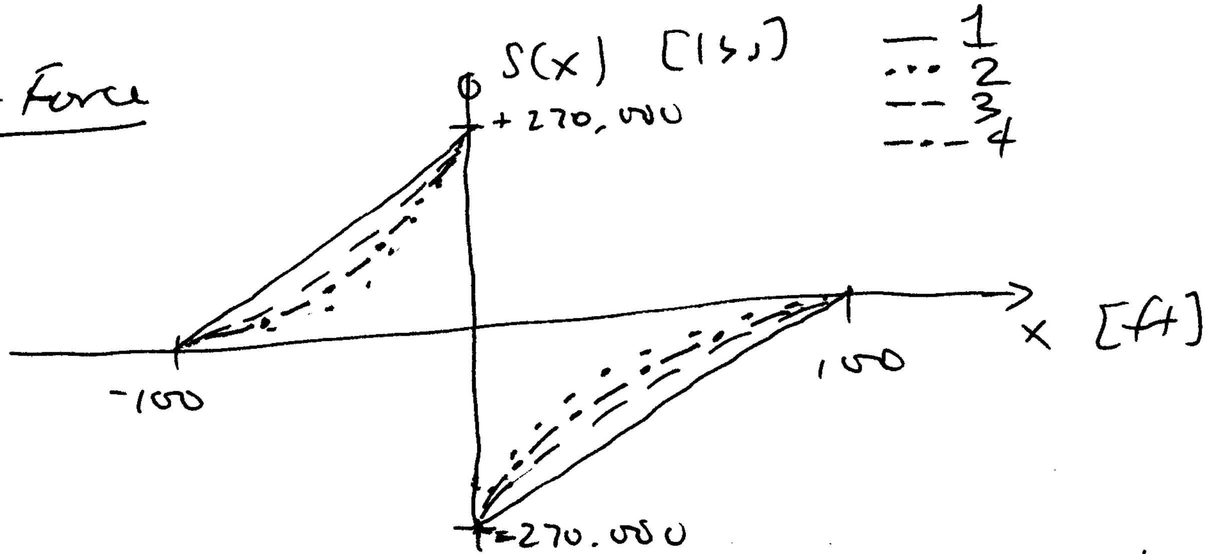
with $q_0 = 4048$ lbs/ft $L = 200$ ft

$P = 540000$ lbs.

(c) Now compare via plotting

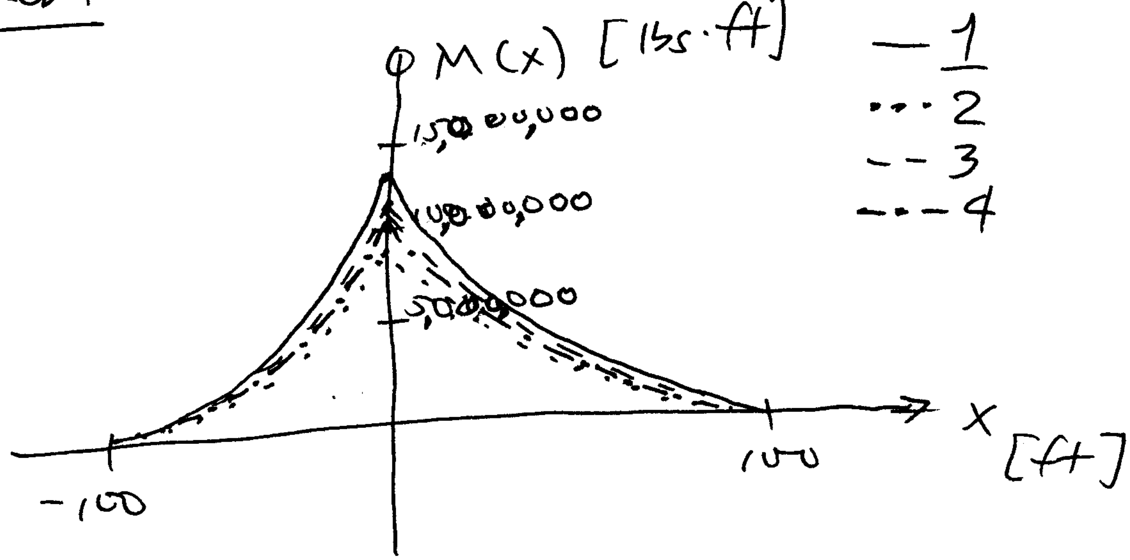
Axial Force -- zero everywhere in all cases (no need to plot)

Shear Force



NOTE: All models have the same value at the root. Reason -- each wing carries the same integrated lift = P/2.
All models change by concentrated weight, P, at the fuselage.

Moment



NOTE: The moment at the root is maximum in all cases, but varies a great deal in value. Thus, the lift distribution plays a considerable role in the moment carried by the beam

(d) The highest moment is at the root, so that is the location of greatest loading as in shear for all cases. The load is transferred at the attachment to the fuselage.